

# Georeferencing of the Third Military Survey of Austrian Monarchy

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**Abstract.** A novel procedure for georeferencing raster images of the Third Military Survey maps from 19th century (1876-1880) is proposed in the contribution. There were several attempts to design a proper transformation model for georeferencing of these maps, but none of them have satisfactorily resolved crucial problems of positional discrepancy between the old maps and contemporary maps (almost 150 m in reality). In the contribution, these problems are approached with the aid of three unique principles.

1. Huge number of control points was utilized to properly match the old maps to the contemporary coordinate system. The set of the control points was validated by means of statistical tests.
2. A complex transformation procedure comprising of four partial sequential transformation steps (rectification of paper shrinkage, reverse projection of a map sheet onto ellipsoid, cartographic projection from the ellipsoid to plane, special elastic 2D transformation). The most important step is the special elastic transformation which corrects inhomogeneous distortions of the old map sheets. Simultaneously, precision of an arbitrary point can be easily estimated.
3. The transformation model is adjustable by a simple set of parameters with comprehensible meaning, namely standard deviation of the permissible positional discrepancy.

Due to these three principles the positional accuracy was reduced to a few meters (9 - 10 m) in reality, which corresponds to 0,4mm on the map sheet. Such a high accuracy is sufficient to produce a seamless map of the Czech Republic composed from the separate transformed map sheets. Therefore, any part of this composition can be compared to some up-to-date map source, eventually to another properly georeferenced old map. Accordingly, the proposed procedure was implemented as a web application on web server of the Research Institute of Geodesy, Topography, and Cartography. Consequently, any Internet user can create overlays of a region of interest from several coverages and compare them to the content of the Third Military Survey maps, e.g. with the aid transparency.

**Keywords:** georeferencing, accuracy analysis, elastic transformation, digital image processing, historical cartography

## 1. Introduction

The motivation for georeferencing raster images of the Third Military Survey maps stems from the ambition to make them available to the professional public on Internet as a web map service (WMS). Broad availability of these old maps enables creating case studies of various branches, namely comparative history, countryside development or urbanism. These studies can greatly benefit from the ability to compare the content of the old maps with contemporary maps or other georeferenced old maps. An example of this comparison is overlaying of the maps with properly chosen transparency. One of the most desired map for this kind of comparison is the Third Military Survey map in 1 : 25 000 scale (so called topographic section) because of high level of detail. If the georeferencing is correctly designed with the aid of a suitable transformation procedure, it should be possible to apply the procedure to the Special Map of the Third Military Survey in 1 : 75 000 scale as well. The Special Map was being used in our cartographic practice for over 70 year and many emissions of these maps were issued. For that reason these maps and the topographic sections are some of the most valuable cartographic sources depicting the development of the Czech countryside and urban centres.

Georeferencing the maps of the Third Military Survey of the Austro-Hungarian Monarchy to contemporary cartographic coordinate system is a long-term problem that has not yet been satisfactorily resolved. This problem concerns primarily map sheets in 1 : 25 000 scale, where the positional differences according to contemporary maps are clearly visible due to their larger scale. Magnitude of these differences are so high that the maps' use in WMS is practically impossible. Common georeferencing techniques, e.g.

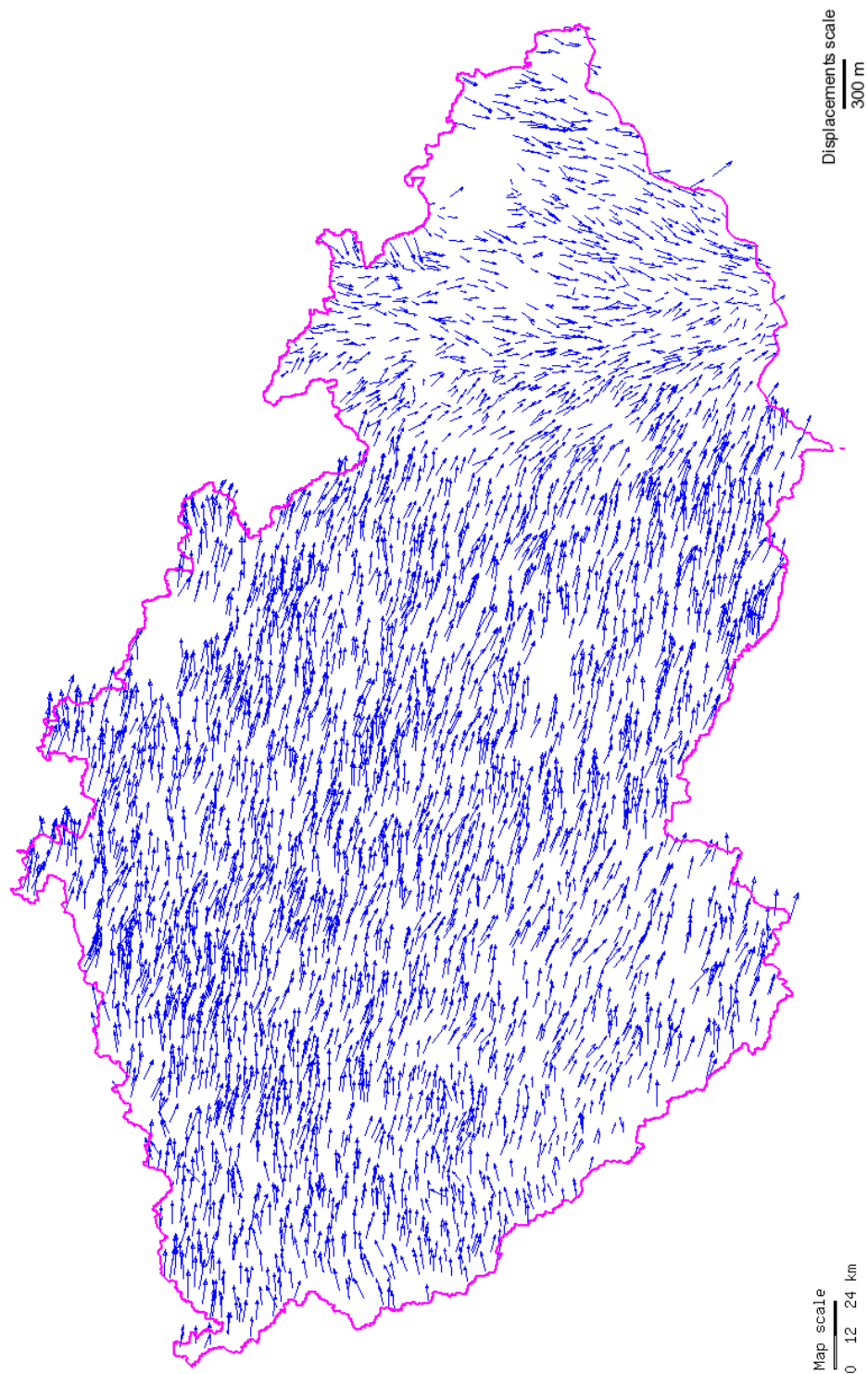
matching only the corners of map sheets, achieve positional errors in range of 12 to 206 m in the extent of the Czech Republic. Such errors are even significantly greater than georeference errors of the the Second Military Survey, which is much older. These errors can be treated as a result of imperfection of the then geodetic control and, simultaneously, shortcomings of the method of the Third Military Survey maps' creation, namely converting insular maps of Stable Cadastre into the topographic sections. A number of papers deal with the analysis of these errors, e.g. (Čechurová 2009), (Čada 2006), (Čada 2005), (Krňoul 2010), (Molnár and Timár 2009), (Molnár and Timár 2011), (Seemann 2008), (Krňoul 2012). The sheer number of works attests to the importance of the problem.

## **2. Positional differences between III. Military Survey and reality**

*Figure 1* shows the positional differences between the maps of III. Military Survey and reality. The arrows represent shifts on a selection of 4 246 trigonometric points obtained by transformation with the use of map sheet corners as control points. As we can see, the shifts from two distinct groups, each with a common global trend - one group covers the Bohemia and western Moravia and the other covers the rest of Moravia. The average shift length is 108 m with standard deviation of 28 m. Most of the shifts, 2370 i.e. 56%, fall into the 90 - 130 m range. The maximum shift length is 206 m.

*Figure 2* shows the consequences of local positional differences when transforming the maps only with the help of map sheet corners. Contemporary National Map 1: 5 000 (deep black outlines of buildings) is overlapped with the transformed III. Military Survey map (gray and colour drawing). The mutual shift is so substantial that visual comparison of both overlapping maps is almost impossible.

Because of these significant problems the best georeference precision achieved to this days has been in the range of several tens of meters (cca 40 m) in reality (see (Čechurová 2009), (Čada 2006), (Krňoul 2010), (Molnár and Timár 2009), (Molnár and Timár 2011), (Seemann 2008), (Krňoul 2012)). It isn't as much important why and how these errors emerged as finding a georeferencing method that would eliminate them as much as possible.



**Figure 1.** Positional errors on trigonometric points in the Third Military Survey



**Figure 2.** Overlap of the III. Military Survey map and contemporary map. Georeferencing was done via transformation with the help of map sheets corners.

### 3. The proposed method of georeferencing

This paper proposes a new method for georeferencing the topographic sections of the III. Military Survey. This method takes advantage of unconventional approaches based on modern statistical methods for experimental data processing. These approaches build on three principles.

1. It is necessary to use as many control points as possible for georeferencing. The validity of the points must be verified by statistical tests.
2. Paper distortion must be rectified and the original map projection must be respected. A special elastic transformation should be used to account for non-homogenous inaccuracy distribution in old maps without causing unacceptable deformations of the maps' content.

3. The parameters of the transformation model should have clear meaning to enable fine-tuning the model to given situation.

The proposed process consists of four steps.

1. Elimination of paper distortion (correcting the sizes of map sheets to original non-distorted size).
2. Projection of the map sheets on Bessel ellipsoid (a projection inverse to the original map projection)
3. Map projection of Bessel ellipsoid into plane (projection into the current map projection of choice, e.g. Křovák's projection). If the selected contemporary map projection is based on other ellipsoid than Bessel's, it is necessary to transform the Bessel ellipsoid coordinates to the appropriate ellipsoid first.
4. Elastic transformation in plane (to correct the non-homogeneous map precision distribution).

Each of these steps is an intermediate transformation. The final transformation is a composition of all these intermediate transformations. The result of application of the final transformation on the source raster images of map sheets is digital images of the maps in the desired coordination system. From these images we can assemble seamless mosaic covering the whole area of the Czech Republic.

## 4. Map distortion elimination

Affine transformation is used to eliminate the map distortion, because the paper is usually shrunk differently in different directions. The decisive factor in this is the manufacturing process of paper that causes the distortion to have extreme values in approximately perpendicular directions.

$$x = QX + r$$

$x$  ... input coordinates

$X$  ... output coordinates

$Q$  ... affine transformation matrix

$r$  ... coordinate system origin offset

The corners of map sheets act as control points. Their coordinates were measured in source raster images. Their coordinates in target coordinate system can be calculated from Bessel ellipsoid parameters with the condi-

tion of preserving the length of central meridian and lateral parallels of four topographic sections (see *chapter 5*).

The affine transformation's equation for control points is:

$$\mathbf{x}_i + \mathbf{v}_i = \mathbf{Q}\mathbf{X}_i + \mathbf{r}$$

$\mathbf{x}_i$  ... input coordinates of control point  $i$

$\mathbf{x}$  ... resulting coordinates of control point  $i$

$\mathbf{v}_i$  ... input coordinates residual of control point  $i$

The affine transformation matrix elements and coordinate system origin offset were estimated by the Least Squares method ( $\hat{\mathbf{Q}}, \hat{\mathbf{r}}$ ). With these parameters known, the transformation formula for the map sheets' scans can be assembled as:

$$\mathbf{X} = \hat{\mathbf{Q}}^{-1}(\mathbf{x} - \hat{\mathbf{r}})$$

## 5. Projecting the map sheets on Bessel ellipsoid

The III. Military Survey uses piecewise Sanson-Flamsteed projection. In this projection, the surface of Bessel ellipsoid is divided into  $30' \times 15'$  (longitude  $\times$  latitude) sections and each section is projected into a plane by means of bilinear projection preserving the length of central meridian and lateral parallels of the section. This results in a trapezoid of appropriate size. The image of the  $30' \times 15'$  section is called section map sheet (in 1 : 75 000 scale) and contains four topographic sections ( in 1 : 25 000 scale). Detailed information about Sanson projection can be found for example in (Boguszak 1961), (Čada 2006), (Seemann 2008), (Krňoul 2012). The area covered by a section sheet on Bessel ellipsoid is obtained by means of inverse Sanson projection. In both directions the transformation can be defined with the help of so called plate. The source coordinates were normalized to  $\langle 0,1 \rangle$  interval.

$$\begin{aligned} \mathbf{p}(T_0, T_1) &= \sum_{i=0}^1 \sum_{j=0}^1 \alpha_{i,j}(T_{1-i}) F_j(T_i) - \mathbf{x}_{i,j} F_i(T_0) F_j(T_1), \\ \alpha_{i,j}(t) &= \mathbf{x}_{j-ij, ij} + (\mathbf{x}_{j-ij+i, ij+1-i} - \mathbf{x}_{j-ij, ij}) \cdot t, \\ F_n(t) &= 1 - n + (2n - 1)t. \end{aligned}$$

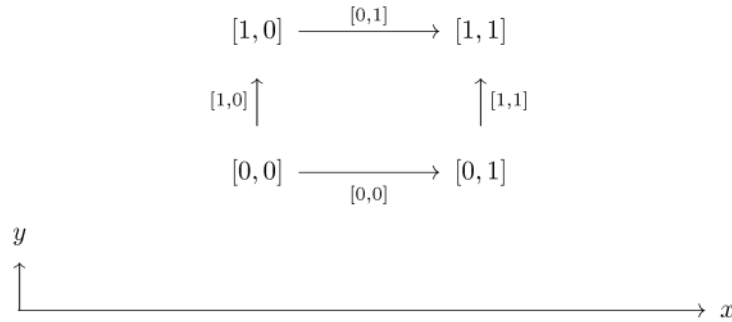
$\mathbf{p}(T_0, T_1)$  ... general point of plate coordinates,  $T_0, T_1 \in \langle 0,1 \rangle$

$\mathbf{x}_{k,l}$  ... coordinates of plate corners

$\alpha_{i,j}$  ... plate edges - line segments between plate corners

$F_n(t)$  ... helper function,  $n \in \{0,1\}, t \in \langle 0,1 \rangle$

The meaning of symbols  $i, j, k, l$  is explained on *Figure 3*.



**Figure 3.** The meaning of indices of corners and edges of map sheets

Indices pairs in corners of the rectangles are indices  $k, l$ . The indices next to the arrows denote the plate edges, i.e. indices  $i, j$ .

To solve the inverse transformation we only need to switch the source and target coordinates systems. However it is always necessary to normalize the source coordinates to the  $\langle 0,1 \rangle$  interval.

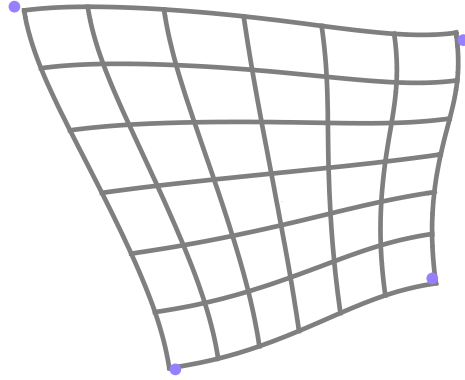
## 6. Map projection of Bessel ellipsoid to plane

We decided to use Křovák's projection, because we wanted results in S-JTSK coordinate system. The projection equations can be found in a number of publications and textbooks, e.g. (Kostelecký et al. 2010). The equations were implemented in custom transformation program.

## 7. Elastic planar transformation

The elastic transformation in the S-JTSK plane was done with the use of collocation. This method allows to find a transformational relation between two coordinate systems that takes into account both the control point precision and non-homogeneous distortion in the coordinate systems. The inhomogeneity causes residual positional deviations on control points after Helmert transformation. The deviations cannot be justified by inaccurate measurement of control points' positions and therefore we can consider them a random variable describing the differences between elastic and similarity transformations. These differences cause nonlinear distortion of equidistant coordinate grid, creating the impression of elasticity (see *Figure 4*).





**Figure 4.** Illustrative example of equidistant coordinate grid distortion of a mapsheet caused by elastic transformation. The control points are at the corners.

These random deviations are statistically dependent. The exact dependency is characterized by covariance matrix of any set of points. The size of non-diagonal elements (so called covariances) diminishes with distance of the points in question. Both factors (control points precision and non-homogeneous distortion of coordinate systems) can be described by intuitive statistical and geometric parameters: standard deviations of errors of control points position determination, standard deviation of difference between elastic and similarity transformations in any point and a characteristic of distances between any two points in which their dependency manifests. The accuracy of the elastic transformation is estimated on the basis of these parameters. Control points are trigonometric points with known S-JTSK coordinates depicted on the map sheets.

The collocation method is based on the following simultaneous equations:

$$\begin{aligned} w' &= p + qw + \varphi \\ w'_i + \varepsilon'_i &= p + q(w_i + \varepsilon_i)\varphi_i \end{aligned}$$

$w$  ... source coordinates

$w'$  ... target coordinates

$p, q$  ... similarity transformation coefficients

$\varphi, \varphi_i$  ... random differences between elastic and similarity transformations

$w$  ... source coordinates of control point  $i$

$w'_i$  ... target coordinates of control point  $i$

$\varepsilon_i, \varepsilon'_i$  ... positional errors of coordinates of control point  $i$

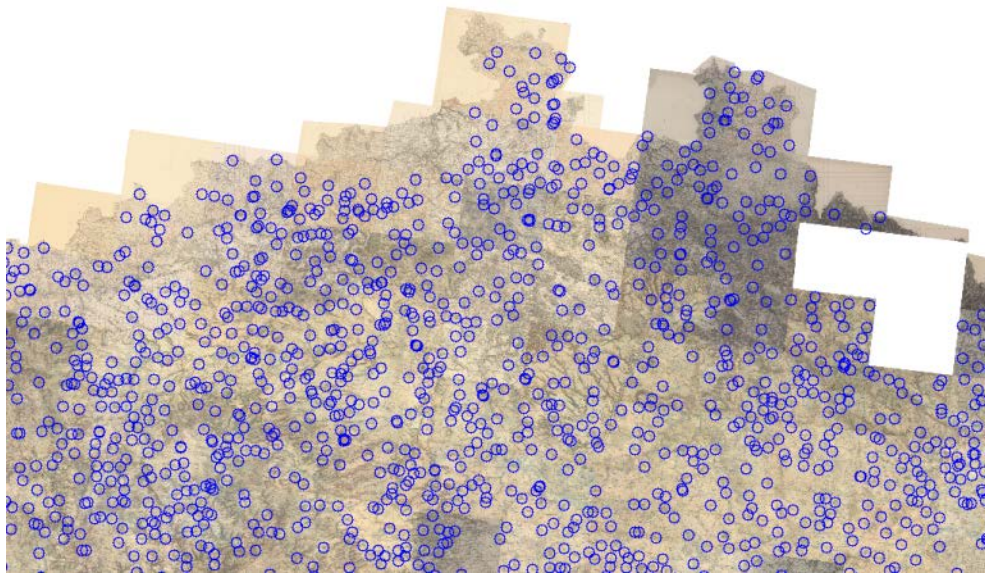
$w', p, q$  ... unknown variables

All of these variables have complex values. Complex arithmetic is also used to calculate the values of unknown variables. This (complex) approach significantly simplifies the computer solution of the simultaneous equations. To solve the inverse transformation we only need to switch the source and target coordination systems.

## 8. The final composite transformation

The final transformation is a composite projection consisting of sequential application of the individual transformations. It is a fairly complex transformation and a demanding calculation which makes it unfeasible to calculate exact values for every pixel. Because of that suitable grid of node points is chosen. Pixel values are computed according the exact equations for these node points. The values for the rest of pixel are obtained via bilinear interpolation. This approach allows to substantially reduce the computing time while preserving the required pixel precision.

The result of the final transformation is a digital image of the source map in contemporary map projection coordinate system (here it is S-JTSK). These images can be arranged into a seamless mosaic covering the whole Czech Republic. An example of this mosaic is on *Figure 5*.



**Figure 5.** Topographic sections mosaic covering the northern part of the Czech Republic. Blue circles represent control points.







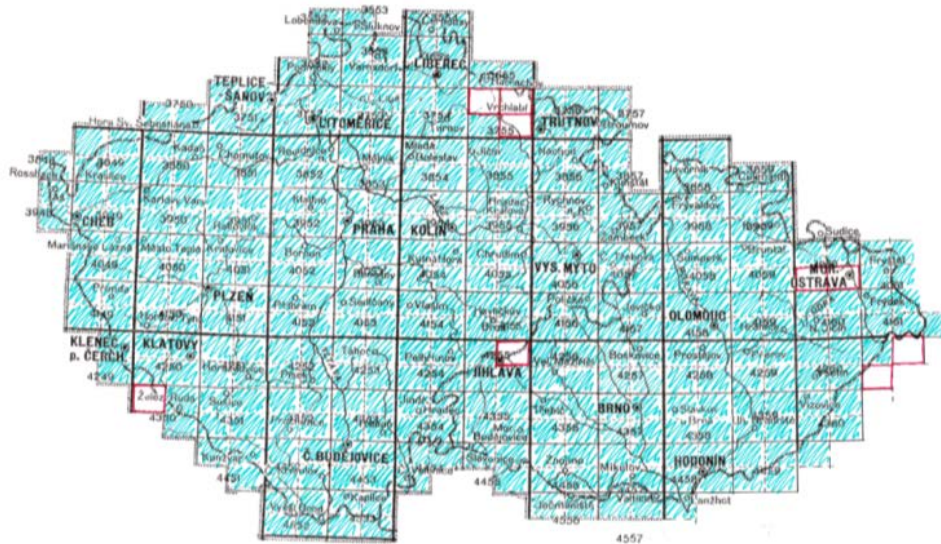
**Figure 7** - Overlap of the III. Military Survey map and contemporary map. Georeferencing was done by the proposed method.

The results of the final transformation can be seen on *Figures 6 and 7*. *Figure 6* shows the connection of four neighbouring map sheets (3851/2, 3852/1, 3851/4, 3852/4). The connection is in the center of a red ellipse. Control points are represented by blue circles and red numbers. The first five digits designate the maps sheet's signature.

*Figure 7* shows an overlay of contemporary vector map data (buildings layer of National Map 1 : 5 000, provided by ČÚZK) over the III. Military Survey map sheet. The contemporary map layer is shown as deep black buildings' outlines over the colour background of the III. Military Survey map. Unlike the situation on *Figure 2*, the two maps here correspond to each other very well.
















## 9. The method for data acquisition

The topographic sections of the III. Military Survey in 1 : 25 000 scale were acquired from several sources to cover as much of the Czech Republic's area as possible. Some of them were found in the map collection of The Institute of History, Academy of Sciences of the Czech Republic, v.v.i. and scanned in VÚGTK, v.v.i. Some of the missing colour originals had to be replaced by black-and-white copies. Out of the 376 topographic sections there is 234 in colour, 133 map sheets are black-and-white and 9 map sheets are still missing. We are still looking for the missing map sheets in various map collections including private collection. The current coverage of the Czech Republic can be seen on *Figure 8*.



**Figure 8.** Overview of scanned topographic sections

To georeference the maps it is necessary to have some control points and obtain images coordinates of map sheets corners. Trigonometric points and churches with known coordinates were selected as control points. Details about the number of various objects' representation in the control points' set is in *Table 1*.

CP type	CP description	Simplified map symbol	Real map symbol (on map)	
			Colour	Black-and-white
TB	Trigonometric point			
K1	Church - type 1			
K2	Church - type 2			
K3	Church - type 3			
K4	Church - type 4			

**Table 1.** Control point types

As many control points as possible were used for each topographic section. Nine sections do not contain any usable control point. These sections depict border areas of the Czech Republic and show only a small part of the Czech territory. The rest of map sheets contain between 1 and 28 control points each. The image coordinates of control points were measured in geodetic program Kokeš. The identification of control points was often very difficult because of the quality of the map print, especially on the black-and-white maps. On some topographic sections even positions of the section corners could not be reliably determined. There were three cases of map sheets with the corners cut away, probably as a result of mistakes and unskilled manipulation with the paper originals. Fifteen sections were reduced to half- or quarter the usual size, likely to save paper when drawing small areas. And finally 6 pairs of topographic sections were joined into a single, larger section.

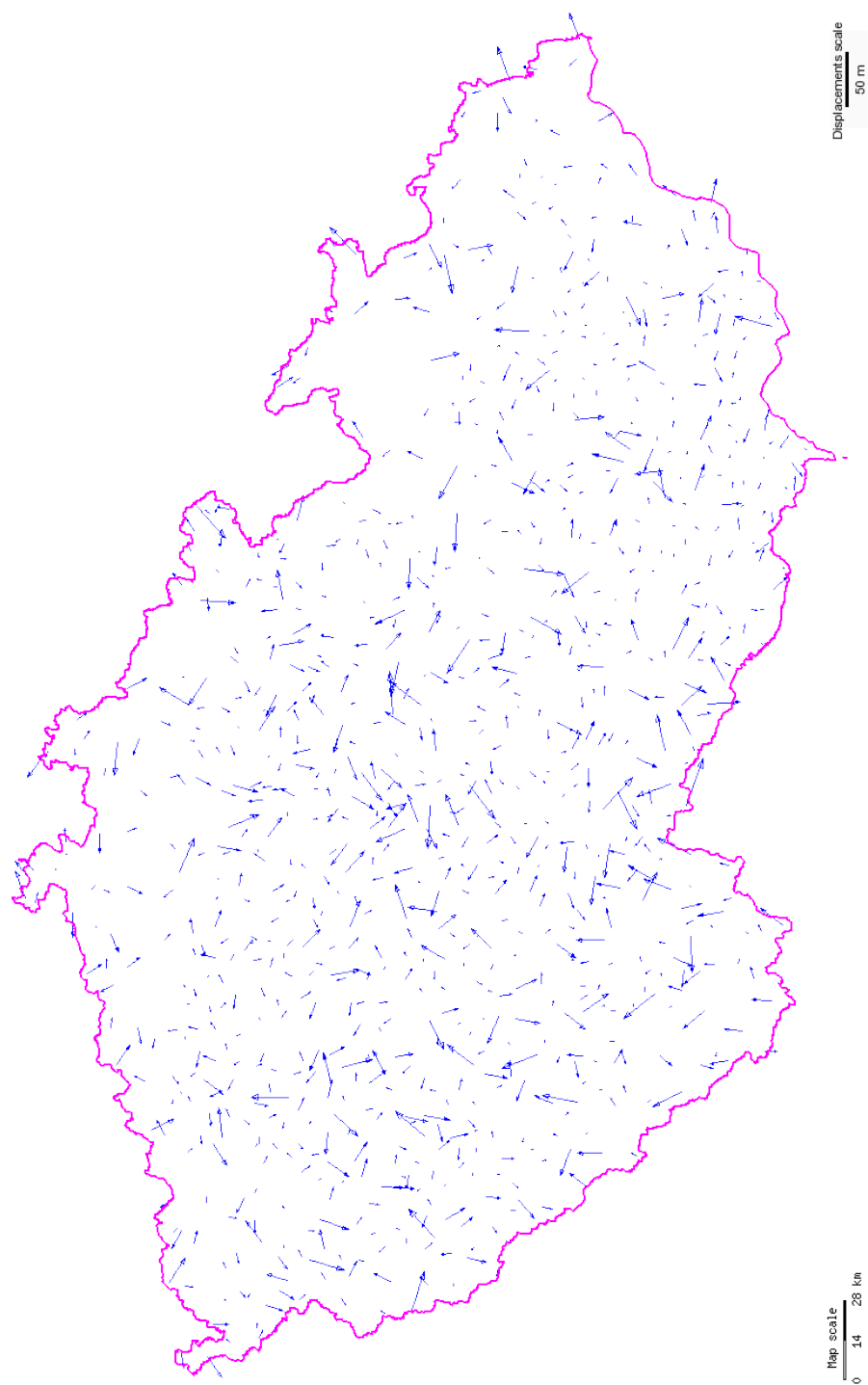
The joined topographic sections: 3654/2 + 3554/4  
3850/1 + 3750/3  
3857/1 + 3857/3  
4455/2 + 4455/4  
4456/4 + 4556/2  
4457/3 + 4557/1

The map shows the administrative districts of the Czech Republic in 1991. Each district is labeled with its name and population. The districts are: Teplice, Litoměřice, Liberec, Trutnov, Hradec Králové, Pardubice, Vrchlabí, Turnov, Mladá Boleslav, Roudnice nad Labem, Chomutov, Kladno, Mělník, Praha, Kolín, Náchod, Rychnov nad Kněžnou, Javorník, Frýdlad, Jihlava, Vyškov, Brno, Olomouc, Znojmo, Hodonín, and Ostrava. The map also shows the borders of the Czech Republic and the surrounding countries: Poland, Germany, and Slovakia. A legend indicates population density: 0-100,000 (white), 100,000-200,000 (light gray), 200,000-300,000 (medium gray), 300,000-400,000 (dark gray), and 400,000-500,000 (black). A scale bar shows distances in kilometers (0, 20, 40, 60, 80, 100).

Number	Corner of the topographic section				Size
	upper left	lower left	upper right	lower right	
3552/4	✓	✓	✓	✓	1/4
3553/3	✓	✓	✓	✓	1/2
3554/3	✗	✗	✗	✓	1/4
3652/2	✓	✓	✓	✓	1/2
3652/3	✓	✓	✓	✓	1/2
3652/4	✗	✓	✓	✓	✓
3653/1	✓	✓	✓	✓	bigger
3653/2	✓	✓	✓	✓	1/4
3655/4	✗	✓	✗	✗	1/8
3756/1	✗	✓	✗	✓	✓
3756/2	✗	✓	✗	✓	1/2
3757/1	✗	✓	✗	✗	1/4
3757/3	✓	✓	✗	✗	✓
3849/1	✓	✓	✓	✓	1/4
3858/1	✗	✗	✗	✓	✓
3858/2	✓	✓	✓	✓	1/4
3858/3	✗	✗	✓	✓	✓
3858/4	✓	✓	✗	✓	✓
3859/3	✗	✓	✗	✓	✓
3859/4	✗	✓	✗	✗	✓
3957/1	✓	✓	✗	✓	✓
3957/2	✗	✓	✗	✓	✓
3958/1	✗	✓	✓	✓	✓
3959/2	✓	✓	✗	✗	✓
3959/4	✓	✓	✗	✗	✓
4060/1	✗	✓	✗	✓	✓
4060/2	✗	✓	✗	✓	1/2
4061/2	✗	✓	✗	✓	✓
4249/2	✗	✓	✓	✓	✓
4455/1	✓	✗	✗	✗	✓
4456/3	✓	✓	✓	✗	✓
4459/4	✓	✗	✓	✓	✓
4552/1	✗	✓	✓	✗	✓
4552/3	✓	✓	✓	✓	1/4
4553/4	✓	✓	✓	✓	1/2
4557/2	✓	✓	✓	✗	✓

**Table 2.** Missing corners and irregular size of topographic sections





**Figure 10.** Positional errors on check points

## 10. Conclusion

The proposed method of georeferencing contributes significantly to the usability of the III. Military Survey maps for practical applications (such as comparative history, countryside development, urbanism, planning, tourism, etc.). This contribution is based on reduction of errors of the old map placement into contemporary coordinate system. It is shown, that the positional discrepancy of map elements that did not change over time can be reduced so much as to make visual comparison possible.

Trigonometric points and church towers were used as control points. The positional accuracy was evaluated with the use of other 958 check points. The positional errors on these points are shown on *Figure 10*. The resulting positional deviation for the check points is 9.1 m. This is a substantial improvement compared to previous georeferencing attempts. This is mainly because of the step 4 of the georeferencing process, i.e. the elastic transformation. The elastic transformation in conjunction with a large quantity of control points allows seamlessly join the individual map sheets, correct the local inaccuracies of the map (deformation and distortion) and properly estimate the final positional accuracy of the transformation. The formal correctness of the accuracy estimation is ensured by the statistical properties of collocation.

The purpose of georeferencing the III. Military Survey was achieved and the results can be published as WMS on server [mapy.vugtk.cz](http://mapy.vugtk.cz). Internet users will be given the ability to use this service in their own applications and studies concerning the study of content of the III. Military Survey maps and its comparison with current reality.

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